

Problem set 10**Due June 6, 2024**

1. Demonstrate that the grand partition function Ξ of a system of non-interacting fermions with two energy levels with energies ϵ_1 and ϵ_2

$$\Xi = \sum_{N=0}^{N_{max}} \lambda^N \sum_{\substack{n_1, n_2 \\ n_1+n_2=N}} \exp[-\beta (n_1\epsilon_1 + n_2\epsilon_2)]$$

where $\beta = 1/k_B T$, $\lambda = \exp(\beta\mu)$, μ being the chemical potential of the system, and N_{max} is determined by the requirement that one fermion can occupy only one microstate, can be expressed as

$$\Xi = [1 + \lambda \exp(-\beta\epsilon_1)] [1 + \lambda \exp(-\beta\epsilon_2)]$$

2. Repeat the derivation for a system of non-interacting bosons with 2 energy levels (note that in this case there is no restriction on N_{max}). The result should be

$$\Xi = [1 - \lambda \exp(-\beta\epsilon_1)]^{-1} [1 - \lambda \exp(-\beta\epsilon_2)]^{-1}$$

Hint: Use the formula for the total sum of a geometric series.