Problem set 9

Due May 23, 2024

1. Based on the Planck law (see the equation below), draw the energy density, $u_\nu(\nu, T) = U_\nu(\nu, T)/V$ of the radiation, as a function of radiation frequency ν, at temperatures $T = 373$ K (boiling water), $T = 1373$ K (burning charcoal) and $T = 1000000$ K (the temperature of the Sun corona). Estimate the most probable wavelength of the radiation at each of those temperature.

$$u_\nu(\nu, T) = \frac{8\pi h\nu^3}{c^3} \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

   where $h = 6.626 \times 10^{-34}$ J×s is the Planck constant, $c = 2.9979 \times 10^8$ m/s is the velocity of light, $k_B = 1.3806 \times 10^{-23}$ J/(mol×K) is the Boltzmann constant, ν is the radiation frequency and T is the absolute temperature. The relationship between the radiation frequency and radiation wavelength (λ) is $\lambda = c/\nu$.

2. Based on the expression for the energy of a canonical ensemble of normal modes with base frequency ν

$$E = \frac{1}{2}h\nu + \frac{h\nu}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

   derive the expression for heat capacity (C_V) of this system at constant volume. Express C_V in units of k_B (i.e., derive the expression for $C_V^* = C_V/k_B$ and in the reduced temperature $T^* = T/(h\nu)$ to obtain an expression independent on the choice of energy and temperature units and of base frequency. Draw the graph $C_V^*(T^*)$.