Problem set 10

Due May 21, 2025

1. Demonstrate that the grand partition function Ξ of a system of non-interacting fermions with two energy levels with energies ϵ_1 and ϵ_2

$$\Xi = \sum_{N=0}^{N_{max}} \lambda^N \sum_{\substack{n_1, n_2 \\ n_1+n_2=N}} \exp\left[-\beta\left(n_1\epsilon_1 + n_2\epsilon_2\right)\right]$$

where $\beta = 1/k_BT$, $\lambda = \exp(\beta\mu)$, μ being the chemical potential of the system, and N_{max} is determined by the requirement that one fermion can occupy only one microstate, can be expressed as

$$\Xi = \left[1 + \lambda \exp\left(-\beta\epsilon_1\right)\right]\left[1 + \lambda \exp\left(-\beta\epsilon_2\right)\right]$$

2. Repeat the derivation for a system of non-interacting bosons with 2 energy levels (note that in this case there is no restriction on N_{max}). The result should be

$$\Xi = \left[1 - \lambda \exp\left(-\beta\epsilon_1\right)\right]^{-1}\left[1 - \lambda \exp\left(-\beta\epsilon_2\right)\right]^{-1}$$

Hint: Use the formula for the total sum of a geometric series.