Problem set 13

Due June 11, 2025

1. Given the expression of the energy of an ideal gas of N boson particles confined to the volume of V at absolute temperature T

$$E(T) = \begin{cases} \frac{3}{2}\frac{k_B T V}{\Lambda(T)^3}\zeta_{5/2}(\lambda(T)) & T \geq T_{crit} \\ \frac{3}{2}\frac{k_B T V}{\Lambda(T)^3}\zeta_{5/2}(1) & T < T_{crit} \end{cases}$$

where $\lambda(T) = \exp\left(\frac{\mu(T)}{k_B T}\right)$ is the fugacity (μ is the chemical potential), k_B is the Boltzmann constant, T_{crit} is the critical Bose-Einstein condensation temperature, Λ is the thermal de Broglie wavelength

$$\Lambda(T) = \frac{h}{\sqrt{2\pi m k_B T}}$$

noindent where m is the mass of the particle and

$$\zeta_r(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^r}$$

derive the expressions for the heat capacity of the gas for $T \geq T_{crit}$ and $T < T_{crit}$.

Make use of the expression for the derivative of $\lambda(T)$ in temperature:

$$\left(\frac{\partial \lambda(T)}{\partial T}\right)_V = -\frac{3}{2}\frac{\lambda(T)}{T}\frac{\zeta_{3/2}(\lambda(T))}{\zeta_{1/2}(\lambda(T))}$$

which can be derived from the relationship between the number density of particles and fugacity (note that the density is fixed).

$$\rho = \frac{N}{V} = \frac{\zeta_{3/2}(\lambda(T))}{\Lambda(T)^3}$$

For clarity, all quantities that depend on temperatures are marked as functions of temperature.

To simplify the derivations, the spin degeneracy, s, is set at 1.