

Problem set 14**Due June 18, 2025**

1. Given the equation of state of the grand canonical system

$$pV = k_B T \ln \Xi$$

and the expansion of the grand partition function Ξ in terms of Z_N 's

$$\Xi = 1 + \sum_{N=1}^{\infty} \frac{Z_N(V, T)}{N!} z^N$$

with

$$z = \lambda Q_1/V$$

$$Z_N = N! \left(\frac{V}{Q_1} \right)^N Q_N = \int \cdots \int \exp(-U_N/k_B T) d\mathbf{r}_1 \cdots d\mathbf{r}_N$$

where λ is the fugacity, Q_N is the partition function of a canonical system with N particles, U_N is the interaction energy for an N -particle system, **derive the expressions for the first 2 coefficients, b_1 and b_2 , in the expansion of pressure in terms of z .**

$$p = k_B T \sum_{j=1}^{\infty} b_j z^j$$

2. Sketch the cluster diagrams corresponding to the following products of the f -functions in the expansion of pressure in terms of density.

$$f_{12}f_{23}f_{34}f_{45}f_{51}f_{14}f_{25}$$

$$f_{12}f_{23}f_{13}f_{34}f_{45}f_{46}f_{56}$$