Problem set 12

Due June 2, 2026

1. Estimate the pressure of an ideal Fermi gas of electrons at 0 K (the zero-point pressure). Find the respective formula in the slides of Lecture 12.

2. The density of sodium metal at room temperature is 0.95 g/cm^3. Assuming that there is one conduction electron per sodium atom, calculate the Fermi temperature of sodium. Find the respective formula in the slides of Lecture 12.

3. Given the expression of the energy of an ideal gas of N boson particles confined to the volume of V at absolute temperature T

$$E(T) = \begin{cases} \frac{3}{2}\frac{k_B T V}{\Lambda(T)^3}\zeta_{5/2}(\lambda(T)) & T \geq T_{crit} \\ \frac{3}{2}\frac{k_B T V}{\Lambda(T)^3}\zeta_{5/2}(1) & T < T_{crit} \end{cases}$$

   where $\lambda(T) = \exp\left(\frac{\mu(T)}{k_B T}\right)$ is the fugacity (μ is the chemical potential), k_B is the Boltzmann constant, T_{crit} is the critical Bose-Einstein condensation temperature, Λ is the thermal de Broglie wavelength

$$\Lambda(T) = \frac{h}{\sqrt{2\pi m k_B T}}$$

   where m is the mass of the particle and

$$\zeta_r(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^r}$$

   derive the expressions for the heat capacity of the gas for $T \geq T_{crit}$ and $T < T_{crit}$.

   Make use of the expression for the derivative of $\lambda(T)$ in temperature:

$$\left(\frac{\partial \lambda(T)}{\partial T}\right)_V = -\frac{3}{2}\frac{\lambda(T)}{T}\frac{\zeta_{3/2}(\lambda(T))}{\zeta_{1/2}(\lambda(T))}$$

which can be derived from the relationship between the number density of particles and fugacity (note that the density is fixed).

$$\rho = \frac{N}{V} = \frac{\zeta_{3/2}(\lambda(T))}{\Lambda(T)^3}$$

For clarity, all quantities that depend on temperatures are marked as functions of temperature.

To simplify the derivations, the spin degeneracy, s, is set at 1.