Problem set 7

Due April 28, 2026

1. Based on the expression for the heat capacity of an ideal monoatomic gas consisting of N particles at constant volume

$$C_V = \frac{3}{2} N k_B$$

derive the expression for the standard deviation of the average energy (σ_E) for an ideal monoatomic gas and for the relative standard deviation of energy (σ_E/E). Compute σ_E/E for 1 picomol (10^{-12} mol) of an ideal gas and comment on the obtained value.

The relationship between energy variance (σ_E^2) and C_V is

$$\sigma_E^2 = k_B T^2 C_V$$

2. Based on the equation of state of an ideal gas (the Clapeyron equation) and the relationship between the variance of density and isothermal compressibility coefficient (κ)

$$\kappa = -\frac{1}{V}\frac{\partial V}{\partial p} = \frac{V}{k_B T \rho^2}\sigma_\rho^2$$

with

$$\rho = \frac{N}{V}$$

where N is the number of atoms, V is the volume and p is the pressure, derive the expression for the relative standard deviation of density, σ_ρ/ρ and show that this quantity is independent of pressure. Calculate σ_ρ/ρ for 1 picomol of an ideal gas and comment on the obtained value.

3. Demonstrate that for a canonical system composed of non-interacting distinguishable sub-systems A, B, $\ldots$, Z, each of the subsystems having the energy levels $E_1^X, E_2^X, \ldots, E_{Z_X}^X$, the partition function Q of

the entire system is the product of the partition functions of the subsystems, Q_A, Q_B, $\ldots$, Q_Z, the the free energy, average energy, entropy, and heat capacity are the sums of the free energies, average energies, entropies, and heat capacities of the subsystems and the pressure is the sum of partial pressures due to the respective subsystems.

4. Based on the Planck law (see the equation below), draw the energy density, $u_\nu(\nu, T) = U_\nu(\nu, T)/V$ of the radiation, as a function of radiation frequency ν, at temperatures $T = 373$ K (boiling water), $T = 1373$ K (burning charcoal) and $T = 1000000$ K (the temperature of the Sun corona). Estimate the most probable wavelength of the radiation at each of those temperature.

$$u_\nu(\nu, T) = \frac{8\pi h\nu^3}{c^3} \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

where $h = 6.626 \times 10^{-34}$ J×s is the Planck constant, $c = 2.9979 \times 10^8$ m/s is the velocity of light, $k_B = 1.3806 \times 10^{-23}$ J/(mol×K) is the Boltzmann constant, ν is the radiation frequency and T is the absolute temperature. The relationship between the radiation frequency and radiation wavelength (λ) is $\lambda = c/\nu$.